Check Your Understanding

13. How are electrostatic force and charge related?

a. The force is proportional to the product of two charges.
b. The force is inversely proportional to the product of two charges.
c. The force is proportional to any one of the charges between which the force is acting.
d. The force is inversely proportional to any one of the charges between which the force is acting.

14. Why is Coulomb's law called an inverse-square law?

a. because the force is proportional to the inverse of the distance squared between charges
b. because the force is proportional to the product of two charges
c. because the force is proportional to the inverse of the product of two charges
d. because the force is proportional to the distance squared between charges

18.3 Electric Field

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Calculate the strength of an electric field
- Create and interpret drawings of electric fields

Section Key Terms

electric field	test charge

You may have heard of a *force field* in science fiction movies, where such fields apply forces at particular positions in space to keep a villain trapped or to protect a spaceship from enemy fire. The concept of a *field* is very useful in physics, although it differs somewhat from what you see in movies.

A *field* is a way of conceptualizing and mapping the force that surrounds any object and acts on another object at a distance without apparent physical connection. For example, the gravitational field surrounding Earth and all other masses represents the gravitational force that would be experienced if another mass were placed at a given point within the field. Michael Faraday, an English physicist of the nineteenth century, proposed the concept of an **electric field**. If you know the electric field, then you can easily calculate the force (magnitude and direction) applied to any electric charge that you place in the field.

An electric field is generated by electric charge and tells us the force per unit charge at all locations in space around a charge distribution. The charge distribution could be a single point charge; a distribution of charge over, say, a flat plate; or a more complex distribution of charge. The electric field extends into space around the charge distribution. Now consider placing a test charge in the field. A **test charge** is a positive electric charge whose charge is so small that it does not significantly disturb the charges that create the electric field. The electric field exerts a force on the test charge in a given direction. The force exerted is proportional to the charge of the test charge. For example, if we double the charge of the test charge, the force exerted on it doubles. Mathematically, saying that electric field is the force per unit charge is written as

$$\vec{E} = \frac{\vec{F}}{q_{\text{test}}} \tag{18.15}$$

where we are considering only electric forces. Note that the electric field is a vector field that points in the same direction as the force on the positive test charge. The units of electric field are N/C.

If the electric field is created by a point charge or a sphere of uniform charge, then the magnitude of the force between this point charge Q and the test charge is given by Coulomb's law

$$F = \frac{k\left|Qq_{\text{test}}\right|}{r^2}$$

where the absolute value is used, because we only consider the magnitude of the force. The magnitude of the electric field is then

$$E = \frac{F}{q_{\text{test}}} = \frac{k\,|Q|}{r^2}. \qquad (18.16)$$

This equation gives the magnitude of the electric field created by a point charge Q. The distance r in the denominator is the distance from the point charge, Q, or from the center of a spherical charge, to the point of interest.

If the test charge is removed from the electric field, the electric field still exists. To create a three-dimensional map of the electric field, imagine placing the test charge in various locations in the field. At each location, measure the force on the charge, and use the vector equation $\vec{E} = \vec{F}/q_{\text{test}}$ to calculate the electric field. Draw an arrow at each point where you place the test charge to represent the strength and the direction of the electric field. The length of the arrows should be proportional to the strength of the electric field. If you join together these arrows, you obtain lines. Figure 18.17 shows an image of the three-dimensional electric field created by a positive charge.

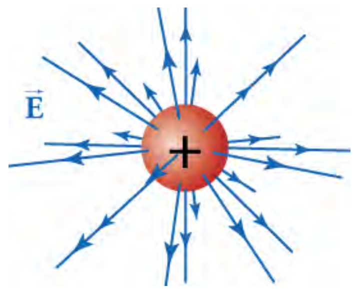


Figure 18.17 Three-dimensional representation of the electric field generated by a positive charge.

Just drawing the electric field lines in a plane that slices through the charge gives the two-dimensional electric-field maps shown in Figure 18.18. On the left is the electric field created by a positive charge, and on the right is the electric field created by a negative charge.

Notice that the electric field lines point away from the positive charge and toward the negative charge. Thus, a positive test charge placed in the electric field of the positive charge will be repelled. This is consistent with Coulomb's law, which says that like charges repel each other. If we place the positive charge in the electric field of the negative charge, the positive charge is attracted to the negative charge. The opposite is true for negative test charges. Thus, the direction of the electric field lines is consistent with what we find by using Coulomb's law.

The equation $E = k\,|Q|\,/r^2$ says that the electric field gets stronger as we approach the charge that generates it. For example, at 2 cm from the charge Q ($r = 2$ cm), the electric field is four times stronger than at 4 cm from the charge ($r = 4$ cm). Looking at Figure 18.17 and Figure 18.18 again, we see that the electric field lines become denser as we approach the charge that generates it. In fact, the density of the electric field lines is proportional to the strength of the electric field!

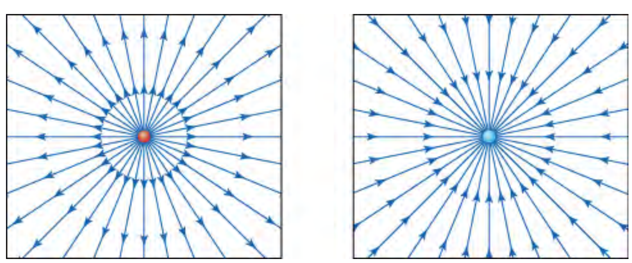


Figure 18.18 Electric field lines from two point charges. The red point on the left carries a charge of +1 nC, and the blue point on the right carries a charge of –1 nC. The arrows point in the direction that a positive test charge would move. The field lines are denser as you approach the point charge.

Electric-field maps can be made for several charges or for more complicated charge distributions. The electric field due to multiple charges may be found by adding together the electric field from each individual charge. Because this sum can only be a single number, we know that only a single electric-field line can go through any given point. In other words, electric-field lines *cannot* cross each other.

Figure 18.19(a) shows a two-dimensional map of the electric field generated by a charge of $+q$ and a nearby charge of $-q$. The three-dimensional version of this map is obtained by rotating this map about the axis that goes through both charges. A positive test charge placed in this field would experience a force in the direction of the field lines at its location. It would thus be repelled from the positive charge and attracted to the negative charge. Figure 18.19(b) shows the electric field generated by two charges of $-q$. Note how the field lines tend to repel each other and do not overlap. A positive test charge placed in this field would be attracted to both charges. If you are far from these two charges, where far means much farther than the distance between the charges, the electric field looks like the electric field from a single charge of $-2q$.

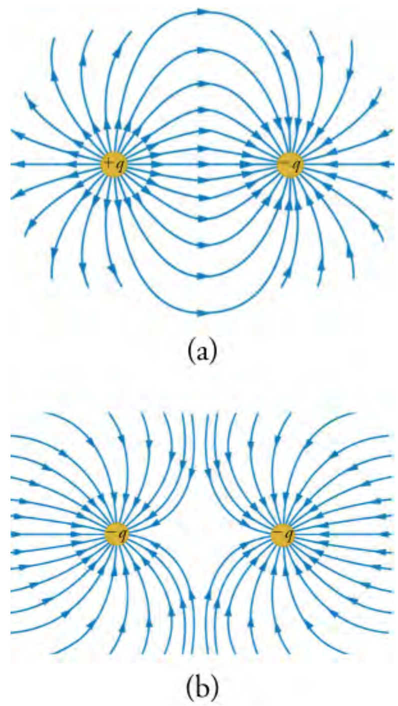


Figure 18.19 (a) The electric field generated by a positive point charge (left) and a negative point charge of the same magnitude (right). (b) The electric field generated by two equal negative charges.

Virtual Physics

Probing an Electric Field

Click to view content (http://www.openstax.org/l/28charge-field)

This simulation shows you the electric field due to charges that you place on the screen. Start by clicking the top checkbox in the options panel on the right-hand side to show the electric field. Drag charges from the buckets onto the screen, move them around, and observe the electric field that they form. To see more precisely the magnitude and direction of the electric field, drag an electric-field sensor, or *E-field* sensor from the bottom bucket, and move it around the screen.

GRASP CHECK

Two positive charges are placed on a screen. Which statement describes the electric field produced by the charges?

a. It is constant everywhere.
b. It is zero near each charge.
c. It is zero halfway between the charges.
d. It is strongest halfway between the charges.

WATCH PHYSICS

Electrostatics (part 2): Interpreting electric field

This video explains how to calculate the electric field of a point charge and how to interpret electric-field maps in general. Note that the lecturer uses d for the distance between particles instead of r. Note that the point charges are infinitesimally small, so all their charges are focused at a point. When larger charged objects are considered, the distance between the objects must be measured between the center of the objects.

Click to view content (https://www.youtube.com/embed/0YOGrTNgGhE)

GRASP CHECK

True or false—If a point charge has electric field lines that point into it, the charge must be ositive.

a. true
b. false

WORKED EXAMPLE

What is the charge?

Look at the drawing of the electric field in Figure 18.20. What is the relative strength and sign of the three charges?

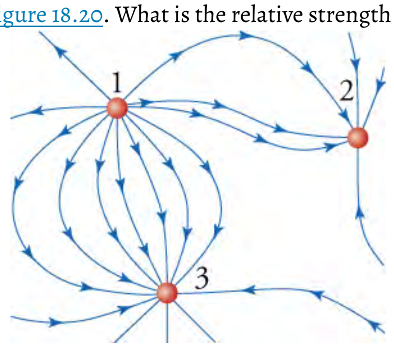


Figure 18.20 Map of electric field due to three charged particles.

STRATEGY

We know the electric field extends out from positive charge and terminates on negative charge. We also know that the number of electric field lines that touch a charge is proportional to the charge. Charge 1 has 12 fields coming out of it. Charge 2 has six field lines going into it. Charge 3 has 12 field lines going into it.

Solution

The electric-field lines come out of charge 1, so it is a positive charge. The electric-field lines go into charges 2 and 3, so they are negative charges. The ratio of the charges is $q_1 : q_2 : q_3 = +12 : -6 : -12$. Thus, magnitude of charges 1 and 3 is twice that of charge 2.

Discussion

Although we cannot determine the precise charge on each particle, we can get a lot of information from the electric field regarding the magnitude and sign of the charges and where the force on a test charge would be greatest (or least).

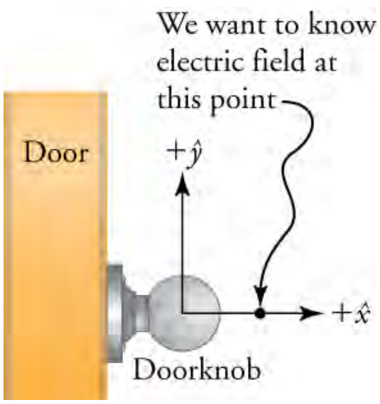
WORKED EXAMPLE

Electric field from doorknob

A doorknob, which can be taken to be a spherical metal conductor, acquires a static electricity charge of $q = -1.5 \text{ nC}$. What is the electric field 1.0 cm in front of the doorknob? The diameter of the doorknob is 5.0 cm.

STRATEGY

Because the doorknob is a conductor, the entire charge is distributed on the outside surface of the metal. In addition, because the doorknob is assumed to be perfectly spherical, the charge on the surface is uniformly distributed, so we can treat the doorknob as if all the charge were located at the center of the doorknob. The validity of this simplification will be proved in a later physics course. Now sketch the doorknob, and define your coordinate system. Use $+x$ to indicate the outward direction perpendicular to the door, with $x = 0$ at the center of the doorknob (as shown in the figure below).



If the diameter of the doorknob is 5.0 cm, its radius is 2.5 cm. We want to know the electric field 1.0 cm from the surface of the doorknob, which is a distance $r = 2.5\text{ cm} + 1.0\text{ cm} = 3.5\text{ cm}$ from the center of the doorknob. We can use the equation $E = \frac{k|Q|}{r^2}$ to find the magnitude of the electric field. The direction of the electric field is determined by the sign of the charge, which is negative in this case.

Solution

Inserting the charge $Q = -1.5\text{ nC} = -1.5 \times 10^{-9}\text{C}$ and the distance $r = 3.5\text{ cm} = 0.035\text{ m}$ into the equation $E = \frac{k|Q|}{r^2}$ gives

$$\begin{aligned} E &= \frac{k|Q|}{r^2} \\ &= \frac{(8.99 \times 10^9 \text{N}\cdot\text{m}^2/\text{C}^2)|-1.5 \times 10^{-9}\text{C}|}{(0.035\ \text{m})^2} \\ &= 1.1 \times 10^4 \text{N/C}. \end{aligned} \quad (18.17)$$

Because the charge is negative, the electric-field lines point toward the center of the doorknob. Thus, the electric field at $x = 3.5\text{ cm}$ is $(-1.1 \times 10^4\text{N/C})\hat{x}$.

Discussion

This seems like an enormous electric field. Luckily, it takes an electric field roughly 100 times stronger ($3 \times 10^6\text{N/C}$) to cause air to break down and conduct electricity. Also, the weight of an adult is about $70\text{ kg} \times 9.8\text{m/s}^2 \approx 700\text{ N}$, so why don't you feel a force on the protons in your hand as you reach for the doorknob? The reason is that your hand contains an equal amount of negative charge, which repels the negative charge in the doorknob. A very small force might develop from polarization in your hand, but you would never notice it.

Practice Problems

15. What is the magnitude of the electric field from 20 cm from a point charge of $q = 33$ nC?

a. 7.4×10^3 N/C
b. 1.48×10^3 N / C
c. 7.4×10^{12} N / C
d. 0

16. A –10 nC charge is at the origin. In which direction does the electric field from the charge point at $x + 10$ cm ?

a. The electric field points away from negative charges.
b. The electric field points toward negative charges.
c. The electric field points toward positive charges.
d. The electric field points away from positive charges.

Check Your Understanding

17. When electric field lines get closer together, what does that tell you about the electric field?

a. The electric field is inversely proportional to the density of electric field lines.
b. The electric field is directly proportional to the density of electric field lines.
c. The electric field is not related to the density of electric field lines.
d. The electric field is inversely proportional to the square root of density of electric field lines.

18. If five electric-field lines come out of a +5 nC charge, how many electric-field lines should come out of a +20 nC charge?
a. five field lines
b. 10 field lines
c. 15 field lines
d. 20 field lines

18.4 Electric Potential

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Explain the similarities and differences between electric potential energy and gravitational potential energy
- Calculate the electric potential difference between two point charges and in a uniform electric field

Section Key Terms

electric potential	electric potential energy

As you learned in studying gravity, a mass in a gravitational field has potential energy, which means it has the potential to accelerate and thereby increase its kinetic energy. This kinetic energy can be used to do work. For example, imagine you want to use a stone to pound a nail into a piece of wood. You first lift the stone high above the nail, which increases the potential energy of the stone-Earth system—because Earth is so large, it does not move, so we usually shorten this by saying simply that the potential energy of the stone increases. When you drop the stone, gravity converts the potential energy into kinetic energy. When the stone hits the nail, it does work by pounding the nail into the wood. The gravitational potential energy is the work that a mass can potentially do by virtue of its position in a gravitational field. Potential energy is a very useful concept, because it can be used with conservation of energy to calculate the motion of masses in a gravitational field.

Electric potential energy works much the same way, but it is based on the electric field instead of the gravitational field. By virtue of its position in an electric field, a charge has an electric potential energy. If the charge is free to move, the force due to the electric field causes it to accelerate, so its potential energy is converted to kinetic energy, just like a mass that falls in a gravitational field. This kinetic energy can be used to do work. The electric potential energy is the work that a charge can do by virtue of its position in an electric field.

The analogy between gravitational potential energy and electric potential energy is depicted in Figure 18.21. On the left, the ball-Earth system gains gravitational potential energy when the ball is higher in Earth's gravitational field. On the right, the two-charge system gains electric potential energy when the positive charge is farther from the negative charge.